? 1.

$$I = \int_{]0,2[} f(x)\,dx.$$

$$1 = \int_0^2 \int_0^{2/\pi} \frac{1}{2}\,\frac{\pi}{2}\,du\,dv.$$

$$\int_0^2 \int_0^{2/\pi} \mathbb{1}(v \leq f(u))\,\frac{1}{2}\,\frac{\pi}{2}\,du\,dv.$$

$$= \frac{1}{2}\,\frac{\pi}{2} \int_0^2 \underbrace{\int_0^{2/\pi} \mathbb{1}(v \leq f(u))\,dv}\,du.$$

$$= \frac{1}{2}\frac{\pi}{2}\int_0^2 f(u)\,du$$

$$= \frac{\pi}{4}\int_0^2 f(u)\,du.$$

$$= \frac{\pi}{4} I.$$

$$I = \frac{4}{\pi}\int_0^2\int_0^{2/\pi} \mathbb{1}(v \leq f(u))\,\frac{1}{2}\frac{\pi}{2}\,du\,dv$$

$$= \frac{4}{\pi}\,\mathbb{E}\left(\mathbb{1}(V \leq f(U))\right)$$

$U \perp\!\!\!\perp V$

$U \sim \mathcal{U}_{[0,2]}$ $V \sim \mathcal{U}_{[0,\frac{2}{\pi}]}$

$$\hat{I} = \frac{4}{\pi}\,\mathrm{prop}\left((U_i, V_i) : V_i \leq f(U_i)\right)$$

$$I = \mathbb{E}\left(\frac{g}{f_X}(X)\right).$$

La LGN s'applique.

Si $\mathbb{E}\left(\left|\frac{g}{f_X}(X)\right|\right) < +\infty.$

$$\left|\frac{g}{f_X}(x)\right| = \frac{|g(x)|}{f_X(x)}$$

$$\mathbb{E}\left(\frac{|g(X)|}{f_X(X)}\right).$$

$$\int_\Delta \frac{|g(x)|}{\cancel{f_X(x)}} \cancel{f_X(x)}\,dx.$$

$$= \int_\Delta |g(x)|\,dx < \infty$$

cf. hyp de départ sur g.