

$$\left(\begin{array}{c} \\ \vdots \\ \end{array} \right) - \frac{1}{r} \left(\begin{array}{c} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_r \end{array} \right) =$$

$$\left(\begin{array}{c} h_{11}x_1x_2\cdots x_n - \alpha_1 \\ \vdots \\ h_{rr}x_1x_2\cdots x_n \end{array} \right) - \left(\begin{array}{c} h_{11}x_1x_2\cdots x_n \\ \vdots \\ h_{rr}x_1x_2\cdots x_n \end{array} \right) \alpha_1 + \frac{1}{r} \left(\begin{array}{c} h_{11}x_1x_2\cdots x_n \\ \vdots \\ h_{rr}x_1x_2\cdots x_n \end{array} \right) =$$

$$\left(\begin{array}{c} \frac{1}{r}x_1x_2\cdots x_n - 1 \\ \vdots \\ \frac{1}{r}x_1x_2\cdots x_n - 1 \end{array} \right) = \left(\begin{array}{c} x_1^T x_2 \cdots x_n \\ \vdots \\ x_1^T x_2 \cdots x_n \end{array} \right) \left(\begin{array}{c} \frac{1}{r} \\ \vdots \\ \frac{1}{r} \end{array} \right) =$$

$$= (x_1^T x_2 \cdots x_n)^T (x_1^T y - n\alpha_1) =$$

$$(x_1^T x_2 \cdots x_n)^T (x_1^T y - n\alpha_1) =$$

$$0 = (x_1^T x_2 \cdots x_n)^T \alpha_1$$

$$= (x_1^T x_2 \cdots x_n)^T \alpha_2$$

$$\cdot \alpha_2 + \cdots + \alpha_2 =$$

$$= \alpha_2 + n x_1^T x_2 \cdots x_n \alpha_2$$

da a $x_1^T (y - x_1^T \alpha_2) = x_1^T (y - x_1^T x_2 \cdots x_n \alpha_2) + (x_1^T x_2 \cdots x_n)^T x_1^T \alpha_2$

et $\alpha_2 = x_1^T (x_1^T x_2 \cdots x_n)^T \alpha_2$ on une CL do colonnes de $x_1^T x_2 \cdots x_n \in \text{Vect}(x_1^T x_2 \cdots x_n)$

mean $y - x_1^T \alpha_2 \in \text{Vect}(x_1^T x_2 \cdots x_n)^{\perp}$ perpendiculaire à $x_1^T x_2 \cdots x_n$

$\langle y - x_1^T \alpha_2, x_1^T x_2 \cdots x_n \rangle = \|y - x_1^T \alpha_2\|^2 + \|n x_1^T \alpha_2\|^2 = 2 \langle y - x_1^T \alpha_2, n x_1^T \alpha_2 \rangle$

$x_1^T \alpha_2 = x_1^T x_2 \cdots x_n (x_1^T x_2 \cdots x_n)^T \alpha_2 = \alpha_2 \rightarrow \text{recte de } x_1^T x_2 \cdots x_n$

$(x_1^T x_2 \cdots x_n)^T \alpha_2 = x_1^T x_2 \cdots x_n (x_1^T x_2 \cdots x_n)^T \alpha_2 = \alpha_2$

$\underbrace{x_1^T x_2 \cdots x_n}_{\alpha_2} - x_1^T x_2 \cdots x_n (x_1^T x_2 \cdots x_n)^T \alpha_2 =$

6. $x(\alpha_2) = x_1^T (x_1^T x_2 \cdots x_n)^T (x_1^T y - n\alpha_1)$

Exercice 2:

1) $\text{corr}(y_i, x_i) = \frac{\sum_{j=1}^n (y_j - \bar{y})(x_j - \bar{x})}{\sqrt{\sum_{j=1}^n (y_j - \bar{y})^2} \sqrt{\sum_{j=1}^n (x_j - \bar{x})^2}}$

2) Si $X_i = \bar{X}$, on sait que pour la solution du problème du moins des écarts au sens des moindres carrés $S = \sum_{i=1}^n (y_i - \hat{y}_i)^2$ est nul. Soit $\hat{y}_i = \bar{y}$. Alors $S = \sum_{i=1}^n (\bar{y} - \bar{y})^2 = 0$.

3) $\lambda \in \mathbb{R}^{1 \times 2} \leftarrow [B(\lambda)]^T = (X^T X)^{-1} (X^T y)$

Il suffit de multiplier par la fonction de la couture pour faire descendre le résultat.

Ensuite, il suffit de calculer $(X^T X)^{-1} (X^T y)$.

0 = $(X^T X)^{-1} (X^T y) = (X^T X)^{-1} (X^T y - n \bar{y})$

Pour $k = 1, 2$, $\lambda_k = \frac{1}{n} \sum_{i=1}^n y_i x_{ik}$ si $x_{ik} \neq 0$.

4) $X_i^T X = \sum_{j=1}^n x_{ij}^2$ pour tout i .

5) $X_i^T (y - X^T \lambda) = \sum_{j=1}^n x_{ij} (y_j - \lambda_j)$

$\lambda_1 = \frac{1}{n} \sum_{j=1}^n x_{1j} (y_j - \lambda_j)$

$\lambda_2 = \frac{1}{n} \sum_{j=1}^n x_{2j} (y_j - \lambda_j)$

$\lambda_3 = \frac{1}{n} \sum_{j=1}^n x_{3j} (y_j - \lambda_j)$

$\lambda_4 = \frac{1}{n} \sum_{j=1}^n x_{4j} (y_j - \lambda_j)$

$\lambda_5 = \frac{1}{n} \sum_{j=1}^n x_{5j} (y_j - \lambda_j)$

$\lambda_6 = \frac{1}{n} \sum_{j=1}^n x_{6j} (y_j - \lambda_j)$

$\lambda_7 = \frac{1}{n} \sum_{j=1}^n x_{7j} (y_j - \lambda_j)$

$\lambda_8 = \frac{1}{n} \sum_{j=1}^n x_{8j} (y_j - \lambda_j)$

$\lambda_9 = \frac{1}{n} \sum_{j=1}^n x_{9j} (y_j - \lambda_j)$

$\lambda_{10} = \frac{1}{n} \sum_{j=1}^n x_{10j} (y_j - \lambda_j)$

$\lambda_{11} = \frac{1}{n} \sum_{j=1}^n x_{11j} (y_j - \lambda_j)$

$\lambda_{12} = \frac{1}{n} \sum_{j=1}^n x_{12j} (y_j - \lambda_j)$

$\lambda_{13} = \frac{1}{n} \sum_{j=1}^n x_{13j} (y_j - \lambda_j)$

$\lambda_{14} = \frac{1}{n} \sum_{j=1}^n x_{14j} (y_j - \lambda_j)$

$\lambda_{15} = \frac{1}{n} \sum_{j=1}^n x_{15j} (y_j - \lambda_j)$

$\lambda_{16} = \frac{1}{n} \sum_{j=1}^n x_{16j} (y_j - \lambda_j)$

$\lambda_{17} = \frac{1}{n} \sum_{j=1}^n x_{17j} (y_j - \lambda_j)$

$\lambda_{18} = \frac{1}{n} \sum_{j=1}^n x_{18j} (y_j - \lambda_j)$

$\lambda_{19} = \frac{1}{n} \sum_{j=1}^n x_{19j} (y_j - \lambda_j)$

$\lambda_{20} = \frac{1}{n} \sum_{j=1}^n x_{20j} (y_j - \lambda_j)$

$\lambda_{21} = \frac{1}{n} \sum_{j=1}^n x_{21j} (y_j - \lambda_j)$

$\lambda_{22} = \frac{1}{n} \sum_{j=1}^n x_{22j} (y_j - \lambda_j)$

$\lambda_{23} = \frac{1}{n} \sum_{j=1}^n x_{23j} (y_j - \lambda_j)$

$\lambda_{24} = \frac{1}{n} \sum_{j=1}^n x_{24j} (y_j - \lambda_j)$

$\lambda_{25} = \frac{1}{n} \sum_{j=1}^n x_{25j} (y_j - \lambda_j)$

$\lambda_{26} = \frac{1}{n} \sum_{j=1}^n x_{26j} (y_j - \lambda_j)$

$\lambda_{27} = \frac{1}{n} \sum_{j=1}^n x_{27j} (y_j - \lambda_j)$

$\lambda_{28} = \frac{1}{n} \sum_{j=1}^n x_{28j} (y_j - \lambda_j)$

$\lambda_{29} = \frac{1}{n} \sum_{j=1}^n x_{29j} (y_j - \lambda_j)$

$\lambda_{30} = \frac{1}{n} \sum_{j=1}^n x_{30j} (y_j - \lambda_j)$

$\lambda_{31} = \frac{1}{n} \sum_{j=1}^n x_{31j} (y_j - \lambda_j)$

$\lambda_{32} = \frac{1}{n} \sum_{j=1}^n x_{32j} (y_j - \lambda_j)$

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$\lambda_{46} = \frac{1}{n} \sum_{j=1}^n x_{46j} (y_j - \lambda_j)$

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$\lambda_{50} = \frac{1}{n} \sum_{j=1}^n x_{50j} (y_j - \lambda_j)$

$\lambda_{51} = \frac{1}{n} \sum_{j=1}^n x_{51j} (y_j - \lambda_j)$

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$\lambda_{100} = \frac{1}{n} \sum_{j=1}^n x_{100j} (y_j - \lambda_j)$