Complex dynamics for the study of cerebral activity in the human brain

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Introduction

- **Motivations**: Study of the relationship between structure and function in the human brain, mimic brain imaging outputs.
- **Local scale objectives**: New ODE model generating cerebral activity (neural, glial, hemodynamic): analysis, validation.
- **Global scale objectives**: Network dynamics analysis.

Local scale

Neural mass model

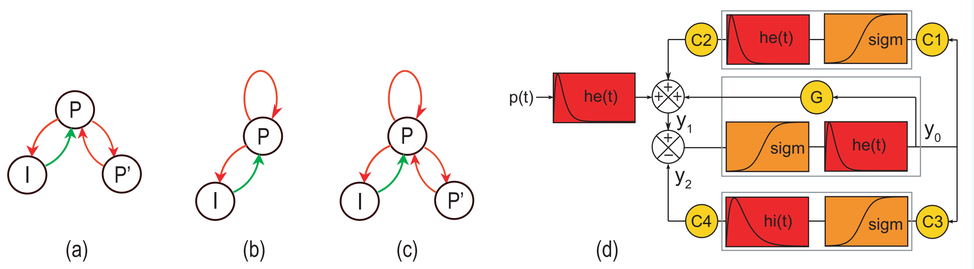


Fig.1: Interactions between neuronal populations: indirect (a), direct (b) and **double (c) and (d)** excitatory feedback from pyramidal cells. P: main pyramidal cells population. P´: intermediary pyramidal cells population. I: Inhibitory interneuron population. Red (resp. green) arrows: excitation (resp. inhibition). $h_e(t)$ ($h_i(t)$): Action potentials $\rightarrow$ excitatory (resp. inhibitory) post-synaptic potential [5]. sigm: average membrane potential $\rightarrow$ average pulse density [2]. C_i ($i \in [\![1,4]\!]$): coupling gains ($C_i = \alpha_i C$, C: maximal number of synaptic connections between two populations) [1]. G: direct feedback coupling gain. $p(t)$: excitatory input representative of interactions with long-range neural populations. State variables: y_0, y_1, y_2.

$$y_0'' = A\,a\,\mathsf{sigm}(y_1 - y_2) - 2\,a\,y_0' - a^2\,y_0$$
$$y_1'' = A\,a\,C_2\,\mathsf{sigm}(C_1\,y_0) - 2\,a\,y_1' - a^2\,y_1 + A\,a\,G\,\mathsf{sigm}(y_1 - y_2) + A\,a\,p(t)$$
$$y_2'' = B\,b\,C_4\,\mathsf{sigm}(C_3\,y_0) - 2\,b\,y_2' - b^2\,y_2$$

Analysis (1)

Bifurcation diagrams according to p and time series glossary

The model can generate five distinct dynamical behaviors:

- Noise Modulated Oscillations (NMO)
- Noise Induced Spiking (NIS)
- Noise Induced Spiking and Over Threshold Oscillations (NIS-OTO)
- Noise Induced Thresholded Amplitude Oscillations (NITAM)
- Noise Induced Spiking and Sub-Threshold Oscillations (NIS-STO)

each one linked to a specific dynamical organisation (Figure 2).

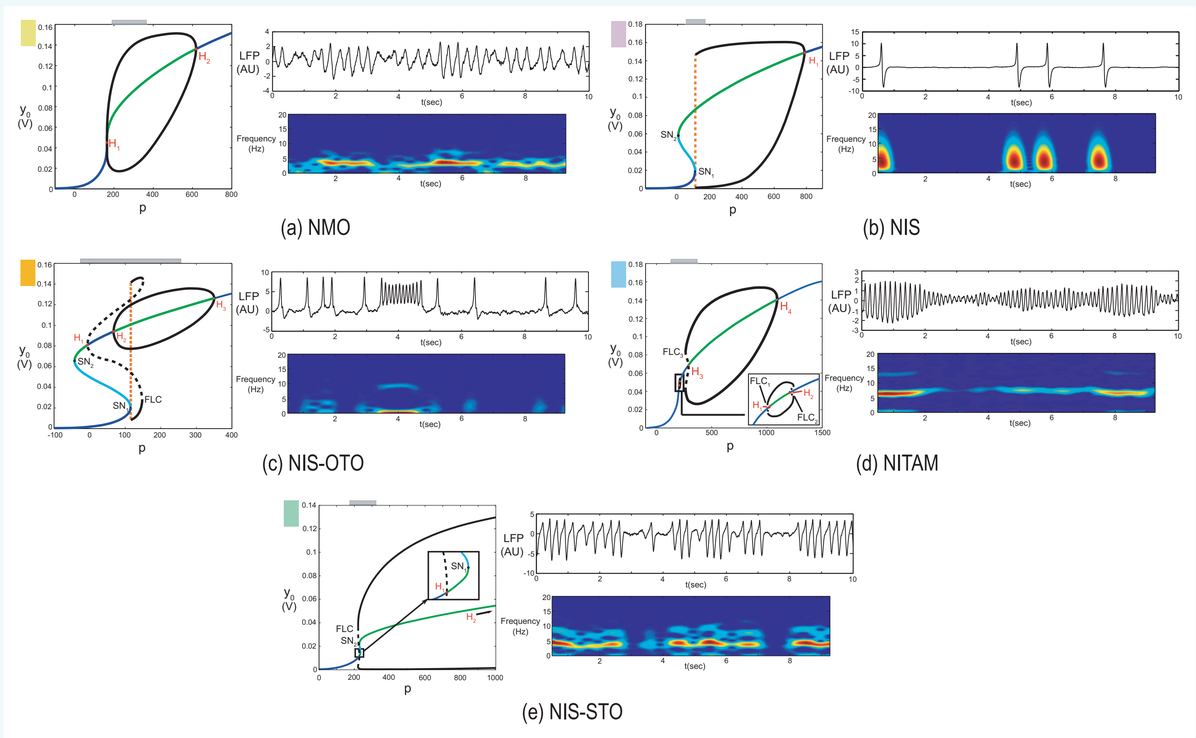


Fig.2: Bifurcation diagrams (left) according to p and instance of associated LFP time series and its spectrogram (right). Blue, cyan and green curves: singular points with 0, 1 and 2 associated eigenvalues with positive real parts respectively. Red points: Hopf bifurcations (H_1, H_2, H_3 and H_4). Black points: saddle-node bifurcations (SN_1 and SN_2). Plain black curves: stable limit cycle extrema. Dashed black curves: unstable limit cycles extrema. Dashed orange line: Saddle-Node on Invariant Circle (SNIC) bifurcation. FLC points: Fold of Limit Cycle bifurcation. [4]

Analysis (2)

Impact of the balance between direct (G) and indirect (α_2) excitatory feedbacks.

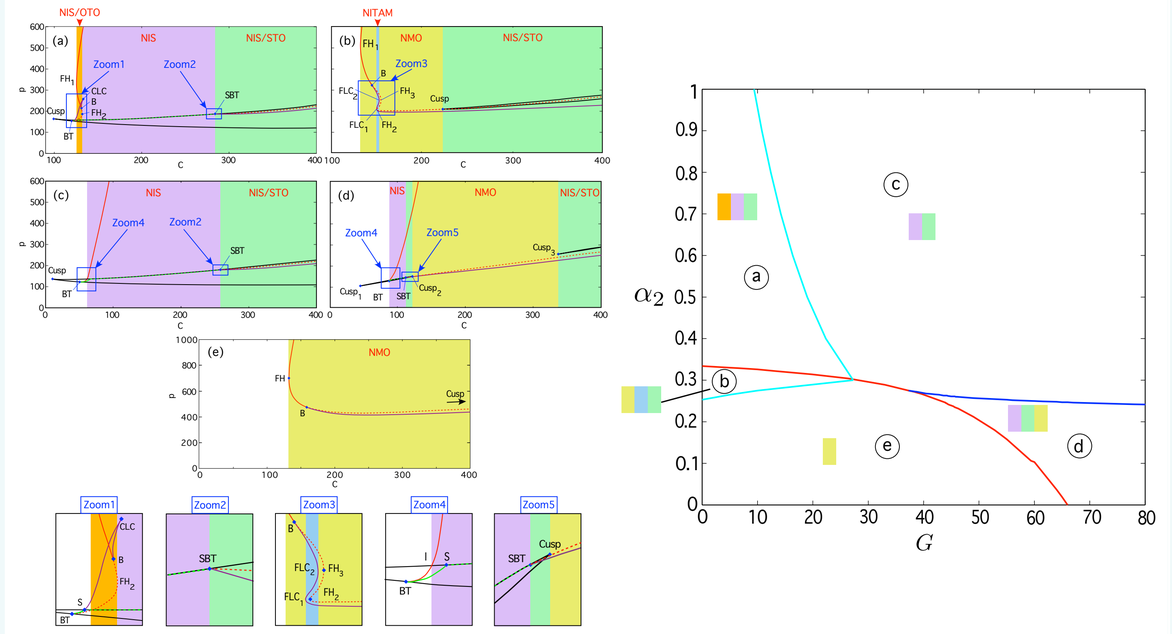
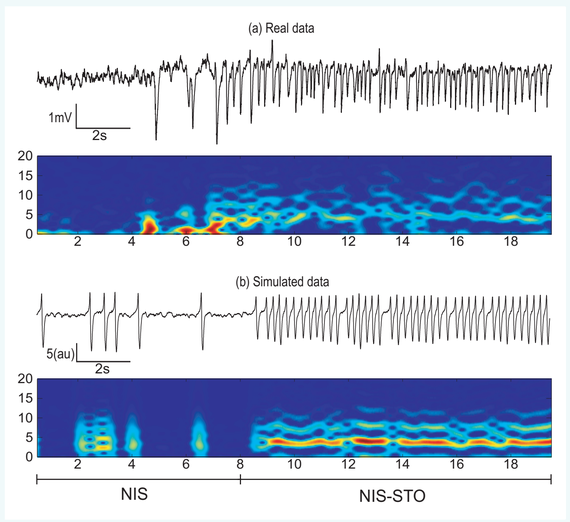


Fig.3: **Left**: Codimension 2 bifurcation diagrams according to C and p. Codimension one bifurcations (curves): saddle-node (black), supercritical Hopf (solid red line), subcritical Hopf (dotted red line), homoclinic connection (green), SNIC (green dots), fold of limit cycles (purple). Codimension two bifurcations (blue diamonds): Cusp, Bogdanov-Takens (BT), Bogdanov-Takens with SNIC (SBT), Bautin (B), Homoclinic connection to SNIC (S), cusp of limit cycles (CLC). Each colored band corresponds to a dynamical behavior: NMO, NIS, NIS-OTO, NITAM and NIS-STO.
Right: Partition of (G, α_2) parameter space according to the type of bifurcation diagrams in (C, p). Cyan curve: appearance/disappearance of two folds of the Hopf bifurcation branch. Red curve: branch of degenerate Bogdanov-Takens bifurcations. Blue curve: branch of cusp/cusp bifurcation. This diagram defines five regions characterized by a single panel of output types: (a): NIS-OTO, NIS, NIS-STO; (b): NMO, NITAM, NIS-STO; (c): NIS, NIS-STO; (d): NIS, NIS-STO, NMO; (e) NMO. [4]

Validation

Comparison with real data



- **Real data**: Hippocampal Discharges (HD) recorded in epileptic mice (Mesial Temporal Lobe Epilepsy (MTLE) mouse model).
- **HD typical features**: sparse large amplitude oscillations followed by lower amplitude rhythmic discharges (ressembling to NIS and NIS-STO behaviors respectively).
- **Time-frequency analysis**: rhytmic discharges of HD and NIS-STO have a same proper frequency of 4 Hz.

Fig.4: Real (a) and simulated (b) time series and their spectrograms. [3]

Perspectives

- **Local scale**: Parameter(s) involved in NIS-STO proper frequency, model reduction, astrocytes and hemodynamic compartments modeling.
- **Network scale**: Local models coupled via anatomical or functional connectivity.

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References

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