Weak discretization error techniques for singular drift SDEs

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Notations

We consider the formal SDE

$$\mathrm{d}X_t = b(t, X_t)\mathrm{d}t + \mathrm{d}Z_t, \qquad X_0 = x \in \mathbb{R}^d, \tag{E}$$

where

- $b \in L^r([0,T], \mathbb{B}^{\beta}_{p,q}(\mathbb{R}^d, \mathbb{R}^d))$ is some *singular* drift.
- Z_t is an α-stable symmetric process $\alpha \in (1,2]$.

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We denote by $\mathcal{L}^\alpha$ the generator of the noise and p_α its density

$$p_\alpha(t,z) := \begin{cases} C_\alpha t^{-\frac{d}{\alpha}} \left(1 + \frac{|z|}{t^{\frac{1}{\alpha}}}\right)^{-(d+\alpha)} & \text{if } \alpha \in (1,2) \\ (2\pi c t)^{-\frac{d}{2}} \exp\left(-\frac{|z|^2}{2ct}\right), \ c \geq 1 & \text{if } \alpha = 2, \end{cases}, \quad t > 0, z \in \mathbb{R}^d$$

Heuristics for well-posedness

For $b \in \mathbb{B}^{\beta}_{\infty,\infty}(\mathbb{R}^d, \mathbb{R}^d) \approx \mathcal{C}^{\beta}$, we work on the PDE

$$(\partial_t + b \cdot \nabla + \mathcal{L}^{\alpha}) u(t,x) = f(t,x) \text{ on } [0,T] \times \mathbb{R}^d \tag{1}$$

where f is some smooth source term.

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Heuristics for well-posedness - $\beta > 0$

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where f is some smooth source term.

- If $\beta > 0$, the condition $\alpha + \beta > 1$ is required to give a meaning to ∇u
  - [CZZ21] : $b \in \mathcal{C}^{\beta}, \alpha \in (0,2)$: weak WP under $\beta + \alpha > 1$.
  - [Pri12] : $b \in \mathcal{C}^{\beta}, \alpha \in (1,2)$: strong WP under $\beta + \frac{\alpha}{2} > 1$.
  - [Por94], [PP95] : $b \in L^p \hookrightarrow \mathbb{B}^{-d/p}_{\infty,\infty}$, $\alpha \in (1,2)$, weak WP under $\frac{d}{p} < \alpha - 1$.
  - [KR05] : $b \in L^q - L^p, \alpha = 2$, strong WP under $\frac{d}{p} + \frac{2}{q} < 1$.

Heuristics for well-posedness - $\beta < 0$

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where f is some smooth source term.

- If $\beta > 0$, the condition $\alpha + \beta > 1$ is required to give a meaning to ∇u
- If $\beta < 0$, in order to make sense of $b \cdot \nabla u$, we need $\beta + (\beta + \alpha - 1) > 0$

$$\text{i.e. } \beta > \frac{1-\alpha}{2}$$

- [FIR17] : $b \in \mathcal{C}^{\beta}, \alpha = 2$, WP (virtual solutions) under $\beta > -\frac{1}{2}$.
- [ABM20] : $b \in \mathcal{C}^{\beta}, \alpha \in (1,2)$, 1D strong WP under $\beta > \frac{1-\alpha}{2}$.
- [CdRM22] : $b \in L^r - \mathbb{B}^{\beta}_{p,q}$, weak WP under

$$2\beta + \alpha - \frac{d}{p} - \frac{\alpha}{r} > 1$$

Discretization of singular drift SDEs

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Aim : approach (E) with

$$X^h_{t_{k+1}} = X^h_{t_k} + hb(t_k, X^h_{t_k}) + (Z_{t_{k+1}} - Z_{t_k}).$$

Or, equivalently, its continuous in time version :

$$\mathrm{d}X^h_t = b(\tau^h_t, X^h_{\tau^h_t})\mathrm{d}t + \mathrm{d}Z_t,$$

where $\tau^h_t = h\lfloor t/h \rfloor$, i.e. if $t \in [t_k, t_{k+1}), \tau^h_t = t_k$.

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- Weak error with a test function :

$$|\mathbb{E}_{0,x}[g(X^h_t) - g(X_t)]|$$

  Weak error on densities (i.e. $g = \delta_y$) :

$$|\Gamma^h(0,x,t,y) - \Gamma(0,x,t,y)|$$

A few important results - Gaussian setting ($\alpha = 2$)

Using PDE techniques :

- [BJ20] $b \in L^\infty$ total variation error : $h^{\frac{1}{2}}$.
- [JM24] $b \in L^q - L^p$ under $\frac{d}{p} + \frac{2}{q} < 1$, weak error : $h^{\frac{1}{2} - \frac{d}{2p} - \frac{1}{q}}$.

Using the sewing lemma ([Lê20]) :

- [LL21] $b \in L^q - L^p$ under $\frac{d}{p} + \frac{2}{q} < 1$, strong error : $h^{\frac{1}{2}} \ln(h)$.
- [DGL22] $b \in L^\infty \cap \dot{W}^\beta_m$, strong error : $h^{\frac{1+\beta}{2} -}$.
- [Hol24] $b \in L^\infty \cap \mathcal{C}^\beta$, weak error on $\mathcal{C}^\beta$ test functions : $h^{\frac{1+\beta}{2} -}$.

Sewing techniques for strong error

In a brownian noise setting, the sewing lemma ([Lê20]) allows to bound expressions of the type

$$\mathbb{E}\left[\left|\int_0^t b(s, X_s^h) - b(s, X_{\tau_s^h}^h)\mathrm{d}s\right|^r\right], \tag{4}$$

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which appear when computing strong error rates.

- Only works for $\alpha = 2$ (also works in a non-Markovian setting).
- Integrability requirements on b.
- The resulting bound *does not* take advantage of the full parabolic bootstrap of the underlying PDE.

Weak error techniques

Let u be a solution to

$$(\partial_s + b(s,x)\cdot\nabla_x + \mathcal{L}^\alpha)\, u(s,x) = 0 \text{ on } [0,t)\times\mathbb{R}^d, \qquad u(t,\cdot) = g \text{ on } \mathbb{R}^d,$$

By Itô's formula,

$$\begin{aligned}\mathcal{E}(g,t,x,h) &= \mathbb{E}_{0,x}[g(X_t^h) - g(X_t)] = \mathbb{E}_{0,x}[u(t,X_t^h) - u(0,x)] \\ &= \mathbb{E}_{0,x}\left[\int_0^t \left(b(r,X_r^h) - b(\tau_r^h, X_{\tau_r^h}^h)\right)\cdot\nabla u(r,X_r^h)\right]\mathrm{d}r.\end{aligned}$$

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Historical approach by [MP91] : for $b \in \mathcal{C}^\beta$

- Schauder estimates for u : $\|\nabla u\|_{L^\infty} < \infty$
- Use the regularity of b to control $\left|b(r,X_r^h) - b(\tau_r^h, X_{\tau_r^h}^h)\right|$

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- Schauder estimates for u : $\|\nabla u\|_{L^\infty} < \infty$
- Use the regularity of b to control $\left|b(r,X_r^h) - b(\tau_r^h, X_{\tau_r^h}^h)\right|$

$$\mathcal{E}(g,t,x,h) \le C\|\nabla u\|_{L^\infty}\int_0^t \mathbb{E}_{0,x}\left[|X_r^h - X_{\tau_r^h}^h|^\beta\right]\mathrm{d}r \le C\|\nabla u\|_{L^\infty} h^{\frac{\beta}{\alpha}}. \qquad (5)$$

Weak error techniques

Compare the Duhamel formulas for Γ^h and Γ :

$$\Gamma^h(0,x,t,y) - \Gamma(0,x,t,y)$$
$$= \mathbb{E}_{0,x}\left[\int_0^t b(s,X_s)\cdot\nabla_y p_\alpha(t-s,y-X_s) - b(\tau_s^h, X^h_{\tau_s^h})\cdot\nabla_y p_\alpha(t-s,y-X_s^h)\mathrm{d}s\right]$$

When b is singular, the main difficulty lies in controlling

$$b(s,X_s) - b(s,X_{\tau_s^h})$$

and

$$b(s,X_s) - b(\tau_s^h, X_s)$$

Weak error techniques - $b(s, X_s) - b(s, X_{\tau_s^h})$

In the splitting of the error, we write the following

$$\mathbb{E}_{0,x}\left[\int_0^t b(s, X_s)\cdot\nabla_y p_\alpha(t-s, y-X_s) - b(s, X_{\tau_s^h})\cdot\nabla_y p_\alpha(t-s, y-X_{\tau_s^h})\mathrm{d}s\right]$$
$$= \int_0^t \int [\Gamma(0, x, s, z) - \Gamma(0, x, \tau_s^h, z)] b(s, z)\cdot\nabla_y p_\alpha(t-s, y-z)\mathrm{d}z\mathrm{d}s$$

Weak error techniques - $b(s, X_s) - b(s, X_{\tau_s^h})$

In the splitting of the error, we write the following

$$\mathbb{E}_{0,x}\left[\int_0^t \left[b(s, X_s)\cdot\nabla_y p_\alpha(t-s, y-X_s) - b(s, X_{\tau_s^h})\cdot\nabla_y p_\alpha(t-s, y-X_{\tau_s^h})\right]\mathrm{d}s\right]$$
$$= \int_0^t \int [\Gamma(0,x,s,z) - \Gamma(0,x,\tau_s^h,z)] b(s,z)\cdot\nabla_y p_\alpha(t-s, y-z)\mathrm{d}z\mathrm{d}s$$

Using bootstrap techniques on the PDE, we can prove

$$|\Gamma(0,x,s,z) - \Gamma(0,x,\tau_s^h,z)| \leq C\frac{(s-\tau_s^h)^{\frac{\gamma}{\alpha}}}{(\tau_s^h)^{\frac{\gamma}{\alpha}}} p_\alpha(\tau_s^h, z-x) \leq C\frac{h^{\frac{\gamma}{\alpha}}}{(\tau_s^h)^{\frac{\gamma}{\alpha}}} p_\alpha(\tau_s^h, z-x)$$

where γ is the *gap to singularity* :

$$\gamma = \alpha - 1 - \frac{d}{p} - \frac{\alpha}{q} > 0 \qquad \text{Lebesgue}, b \in L^q - L^p$$
$$\gamma = \alpha + \beta - 1 > 0 \qquad \text{Holder}, b \in \mathcal{C}^\beta$$
$$\gamma = \beta + \frac{\alpha - 1 - \frac{d}{p} - \frac{\alpha}{q}}{2} > 0 \qquad \text{Negative Besov}, b \in L^q - \mathbb{B}^\beta_{p,r}$$

Weak error techniques - $b(s, X_s) - b(s, X_{\tau_s^h})$

It remains to show the boundedness of

$$\int_0^t \int p_\alpha(s, z - x) b(s, z) \nabla_y p_\alpha(t - s, y - z) \mathrm{d}z \mathrm{d}s.$$

If $b \in \mathfrak{H}$ in space, we have to estimate

$$\| p_\alpha(s, \cdot - x) \nabla_y p_\alpha(t - s, y - \cdot) \|_{\mathfrak{H}^*},$$

and recoup singularities in s and $t - s$, which we want to integrate against $\| b(s, \cdot) \|_{\mathfrak{H}}$.

Weak error techniques - $b(s, X_s) - b(\tau_s^h, X_s)$

Natural time-space scaling :
When $b(t, \cdot) \in \mathcal{C}^\beta$ in space, we usually assume $b(\cdot, z) \in \mathcal{C}^{\frac{\beta}{\alpha}}$, i.e.

$$|b(s, X_s) - b(\tau_s^h, X_s)| \leq (s - \tau_s^h)^{\frac{\beta}{\alpha}}$$

Not enough to show a convergence with order

$$\frac{\gamma}{\alpha} = \frac{\beta}{\alpha} + \frac{\alpha - 1}{\alpha}$$

Weak error techniques - $b(s, X_s) - b(\tau_s^h, X_s)$

Time-randomized Euler scheme :

$$X^h_{t_{k+1}} = X^h_{t_k} + hb(U_k, X^h_{t_k}) + (Z_{t_{k+1}} - Z_{t_k}),$$

where $(U_k) \sim \mathcal{U}([t_k, t_{k+1}])$.
Duhamel integral over one time-step for the time-randomized Euler scheme :

$$\begin{aligned}
&\int_{t_k}^{t_{k+1}} \mathbb{E}\left[b(U_k, X^h_{\tau^h_s}) \cdot \nabla_y p_\alpha(t - U_k, y - X^h_{\tau^h_s})\right] \mathrm{d}s \\
&\quad = \int_{t_k}^{t_{k+1}} \frac{1}{h} \int_{t_k}^{t_{k+1}} \mathbb{E}\left[b(r, X^h_{\tau^h_s}) \cdot \nabla_y p_\alpha(t - r, y - X^h_{\tau^h_s})\right] \mathrm{d}r\mathrm{d}s \\
&\quad = \int_{t_k}^{t_{k+1}} \frac{1}{h} \int_{t_k}^{t_{k+1}} \mathbb{E}\left[b(r, X^h_{\tau^h_r}) \cdot \nabla_y p_\alpha(t - r, y - X^h_{\tau^h_r})\right] \mathrm{d}r\mathrm{d}s \\
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\end{aligned}$$

Results - Lebesgue case, $\alpha \in (1, 2)$

Theorem ([F., Jourdain, Menozzi, 2024])

For $b \in L^q - L^p$, if

$$\gamma = \alpha - 1 - \frac{d}{p} - \frac{\alpha}{q} > 0,$$

then equation

$$\mathrm{d}X_t = b(t, X_t)\mathrm{d}t + \mathrm{d}Z_t, \qquad X_0 = x \in \mathbb{R}^d$$

is weakly well-posed (in the sense of the martingale problem).

Theorem ([F., Jourdain, Menozzi, 2024])

For $b \in L^q - L^p$, we define the Euler scheme by

$$X^h_{t_{k+1}} = X^h_{t_k} + hb_h(U_k, X^h_{t_k}) + (Z_{t_{k+1}} - Z_{t_k}),$$

where b_h is a cut-offed b s.t. $|b_h| \leq h^{-\frac{d}{\alpha p} - \frac{1}{q}}$ and $\forall k, U_k \sim \mathcal{U}(t_k, t_{k+1})$. Then,

$$|\Gamma(0, x, t, y) - \Gamma^h(0, x, t, y)| \leq Ch^{\frac{\gamma}{\alpha}} p_\alpha(t, y - x) \tag{6}$$

Results - Hölder case , $\alpha \in (1,2]$

Theorem ([F., Menozzi, 2024])

For $b \in L^\infty - \mathcal{C}^\beta, \beta > 0$, we define the Euler scheme by

$$X^h_{t_{k+1}} = X^h_{t_k} + hb(U_k, X^h_{t_k}) + (Z_{t_{k+1}} - Z_{t_k}),$$

where $\forall k, U_k \sim \mathcal{U}(t_k, t_{k+1})$. Define

$$\gamma = \beta + \alpha - 1 > 0.$$

Then,

$$|\Gamma(0,x,t,y) - \Gamma^h(0,x,t,y)| \leq Ch^{\frac{\gamma}{\alpha}}(1 + t^{-\frac{\beta}{\alpha}})p_\alpha(t, y-x) \tag{7}$$

Besov case as a consequence of the Hölder case

- Hölder : $|\Gamma(0,x,t,y) - \Gamma^h(0,x,t,y)| \le C[b]_{\mathcal{C}^\beta} h^{\frac{\beta+\alpha-1}{\alpha}} p_\alpha(t, y-x)$
- Negative Besov regularity : let $b \in L^r([0,T], \mathbb{B}^\eta_{p,q}), \eta < 0$ and set

$$\mathfrak{b}(s,z,h) := \frac{1}{h}\int_s^{s+h}\int b(u,y) p_\alpha(u-s, y-z)\mathrm{d}y\mathrm{d}u.$$

Then, $\mathfrak{b}(s,\cdot,h)$ is β-Hölder with $\beta > 0$ and its Hölder modulus

$$|\mathfrak{b}(s,z,h) - \mathfrak{b}(s,z',h)| \le C|z-z'|^\beta h^{-\frac{1}{r}-\frac{\beta}{\alpha}+\frac{\eta}{\alpha}-\frac{d}{\alpha p}}$$

We then apply the Hölder result to the SDE with drift $\mathfrak{b}$, and we get

$$\begin{aligned}|\Gamma(0,x,t,y) - \Gamma^h(0,x,t,y)| &\le Ch^{\frac{\beta+\alpha-1}{\alpha}} h^{-\frac{1}{r}-\frac{\beta}{\alpha}+\frac{\eta}{\alpha}-\frac{d}{\alpha p}} p_\alpha(t,y-x) \\ &\le Ch^{\frac{\eta+\alpha-\frac{\alpha}{r}-\frac{d}{p}-1}{\alpha}} p_\alpha(t,y-x)\end{aligned}$$

Dynamics of the SDE

Proposition ([DD16],[CdRM22])

Assume

$$\alpha \in \left(\frac{1+\frac{d}{p}}{1-\frac{1}{r}}, 2 \right) \qquad \beta \in \left(\frac{1-\alpha+\frac{2d}{p}+\frac{2\alpha}{r}}{2}, 0 \right), \tag{GRD}$$

Then, the formal SDE

$$\mathrm{d}X_t = b(t, X_t)\mathrm{d}t + \mathrm{d}Z_t, \qquad X_0 = x \in \mathbb{R}^d$$

rewrites

$$X_t = X_0 + \int_0^t \mathfrak{b}(s, X_s, \mathrm{d}s) + Z_t,$$

where $\forall 0 \leq v \leq s \leq T$,

$$\mathfrak{b}(v, x, s-v) = \int_v^s \int b(r, y) p_\alpha(r-v, y-x)\mathrm{d}y\mathrm{d}r$$

Thank you for your attention.

Siva Athreya, Oleg Butkovsky, and Leonid Mytnik.
Strong existence and uniqueness for stable stochastic differential equations with distributional drift.
Ann. Probab., 48(1) :178–210, 2020.

Oumaima Bencheikh and Benjamin Jourdain.
Convergence in total variation of the euler-maruyama scheme applied to diffusion processes with measurable drift coefficient and additive noise, 2020.

Paul-Éric Chaudru de Raynal and Stéphane Menozzi.
On multidimensional stable-driven stochastic differential equations with Besov drift.
Electronic Journal of Probability, 27(none) :1 – 52, 2022.

Zhen-Qing Chen, Xicheng Zhang, and Guohuan Zhao.
Supercritical SDEs driven by multiplicative stable-like Lévy processes.
Trans. Amer. Math. Soc., 374(11) :7621–7655, 2021.

François Delarue and Roland Diel.
Rough paths and 1d SDE with a time dependent distributional drift : application to polymers.
Probability Theory and Related Fields, 165(1) :1–63, Jun 2016.

Konstantinos Dareiotis, Máté Gerencsér, and Khoa Le.
Quantifying a convergence theorem of gyöngy and krylov, 2022.

Franco Flandoli, Elena Issoglio, and Francesco Russo.
Multidimensional stochastic differential equations with distributional drift.
Transactions of the American Mathematical Society, 369 :1665–1688, 2017.

Bibliographie II

Teodor Holland.
A note on the weak rate of convergence for the Euler-Maruyama scheme with Hölder drift.
Stoch. Proc and Appl., 174, paper #104379, 2024.

Benjamin Jourdain and Stéphane Menozzi.
Convergence Rate of the Euler-Maruyama Scheme Applied to Diffusion Processes with $L^q - L^\rho$ Drift Coefficient and Additive Noise.
Annals of Applied Probability, 34–1b :1663–1697, 2024.

Nikolay Krylov and Michael Röckner.
Strong solutions of stochastic equations with singular time dependent drift.
Probab. Theory Related Fields, 131(2) :154–196, 2005.

Khoa Lê.
A stochastic sewing lemma and applications.
Electronic Journal of Probability, 25(none) :1 – 55, 2020.

Khoa Le and Chengcheng Ling.
Taming singular stochastic differential equations : A numerical method, 2021.

Remigijus Mikulevicius and Eckhard Platen.
Rate of convergence of the Euler approximation for diffusion processes.
Mathematatische Nachrichten, 1991.

N. I. Portenko.
Some perturbations of drift-type for symmetric stable processes.
Random Oper. Stochastic Equations, 2(3) :211–224, 1994.

S. I. Podolynny and N. I. Portenko.
On multidimensional stable processes with locally unbounded drift.
Random Oper. Stochastic Equations, 3(2) :113–124, 1995.

Bibliographie III

Enrico Priola.
Pathwise uniqueness for singular SDEs driven by stable processes.
Osaka Journal of Mathematics, pages 421 – 447, 2012.